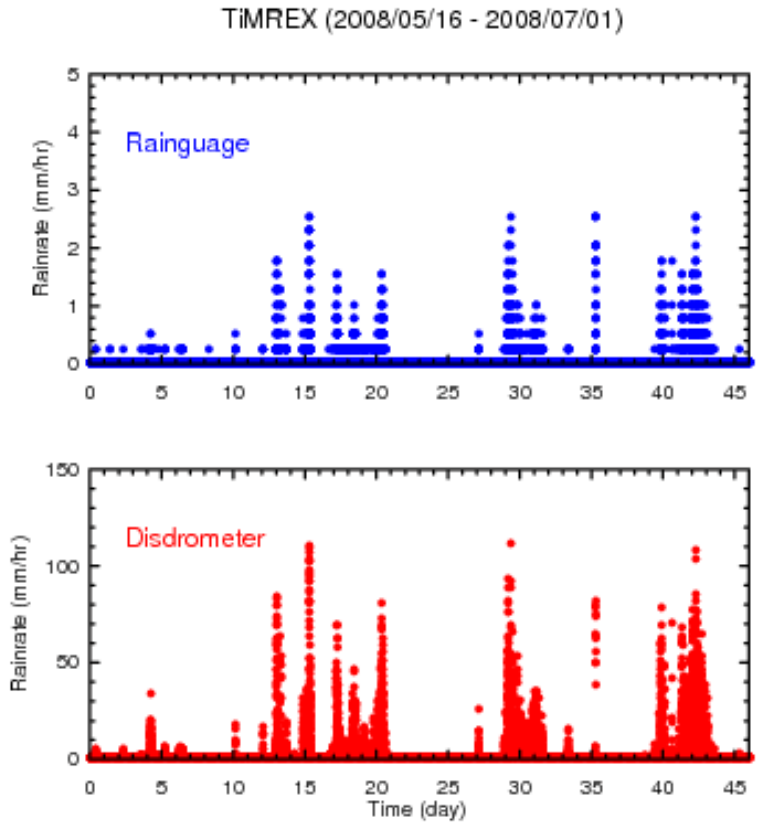
Scale Invariance of Drop-Size Concentration (SoWMEX/TiMREX 08)

H.-m. Hsu[1], P.-L. Lin[2], C.-S. Chen[2] and W.-Y. Chang[2]

[1]NCAR, [2]NCU



- To compare rain-rate time series between disdrometer and raingauge
- To characterize them ***statistically***

Scale Invariance:

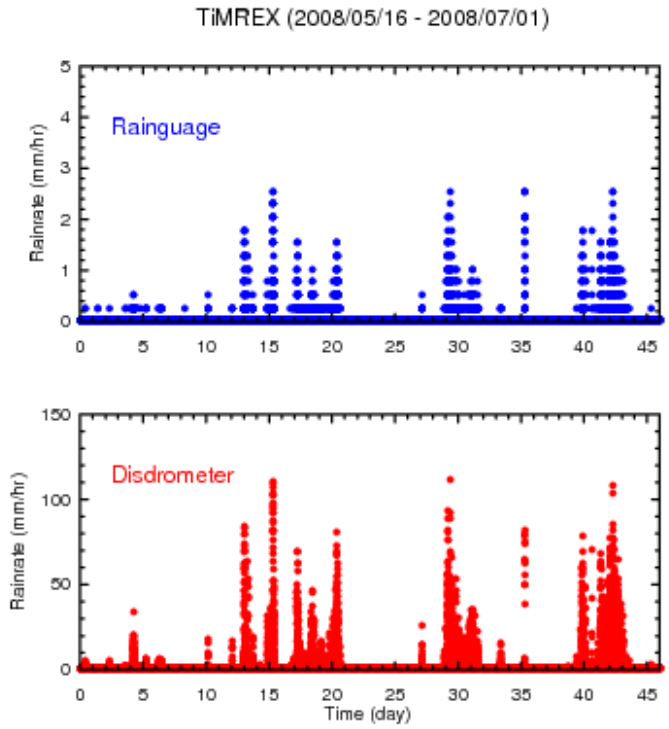
$$F(\lambda x) = \lambda^{\pm\xi} F(x)$$

Self-similarity

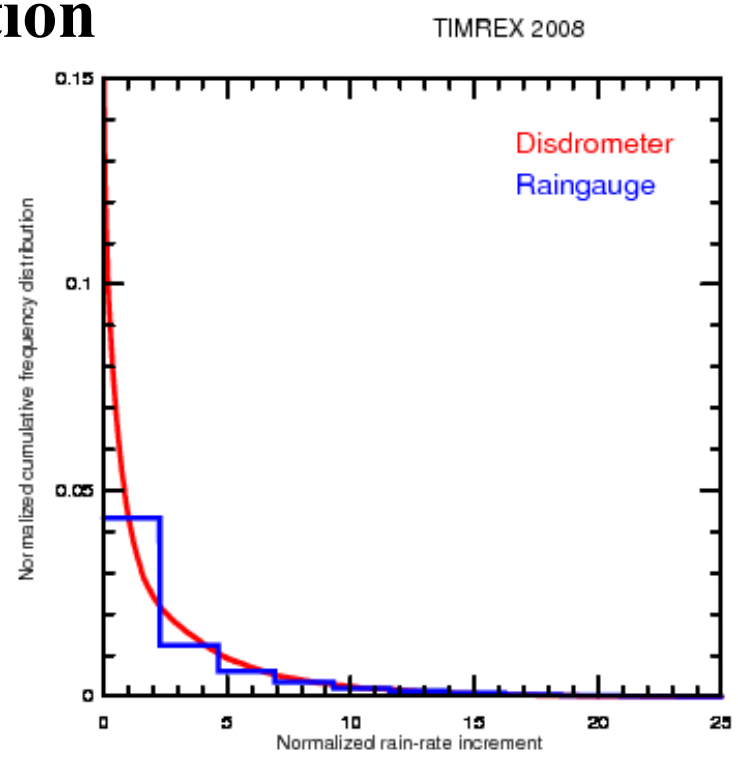
Power-Law Scaling:

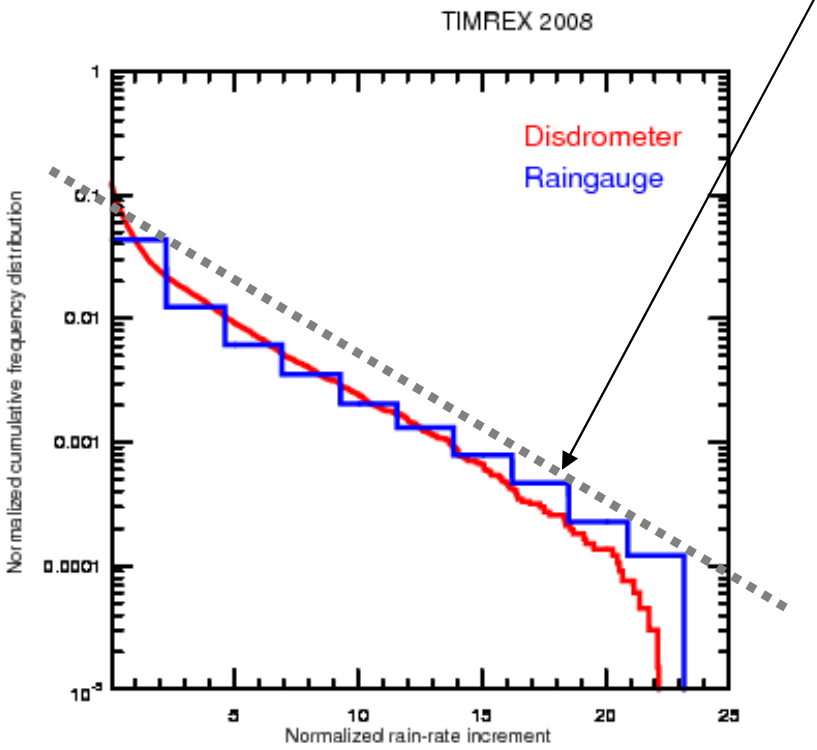
$$F(x) = x^{\pm\beta}$$

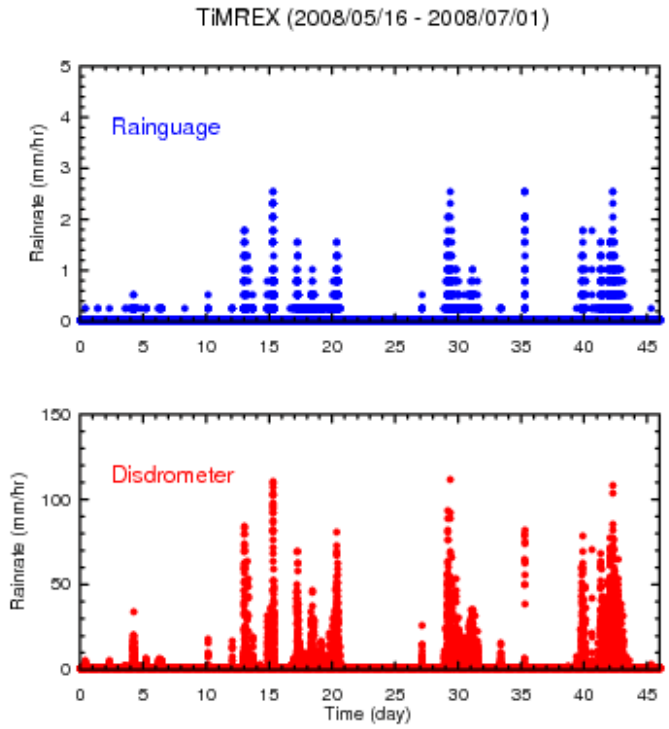
Cumulative Frequency Distribution



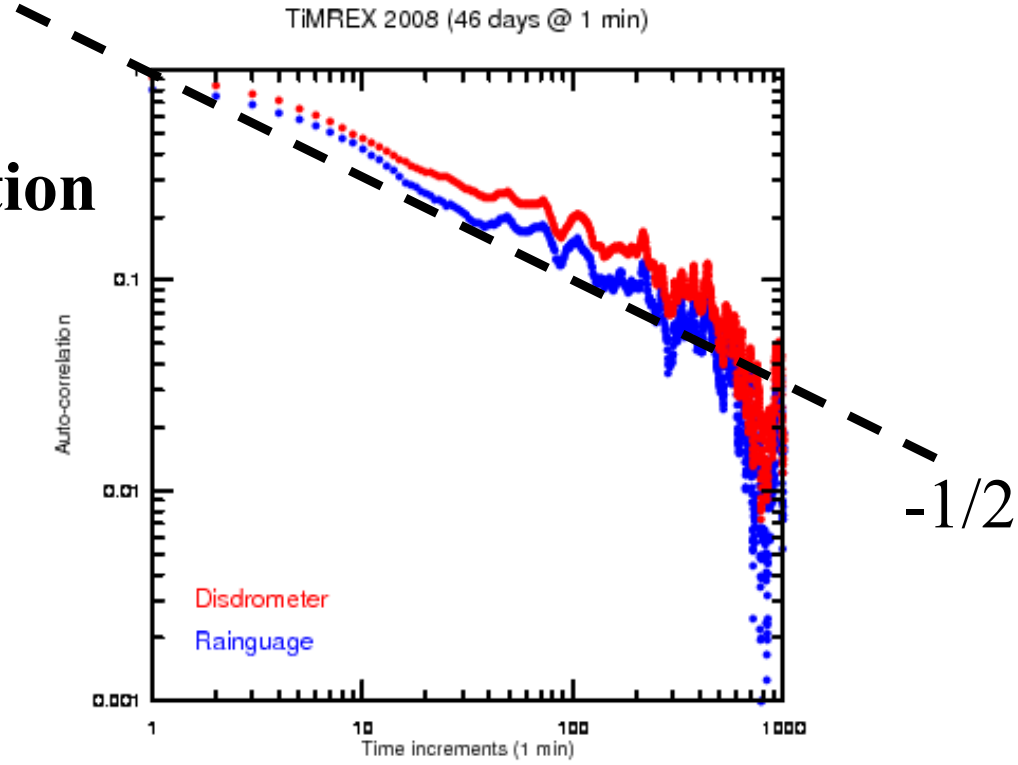
Exponential distribution

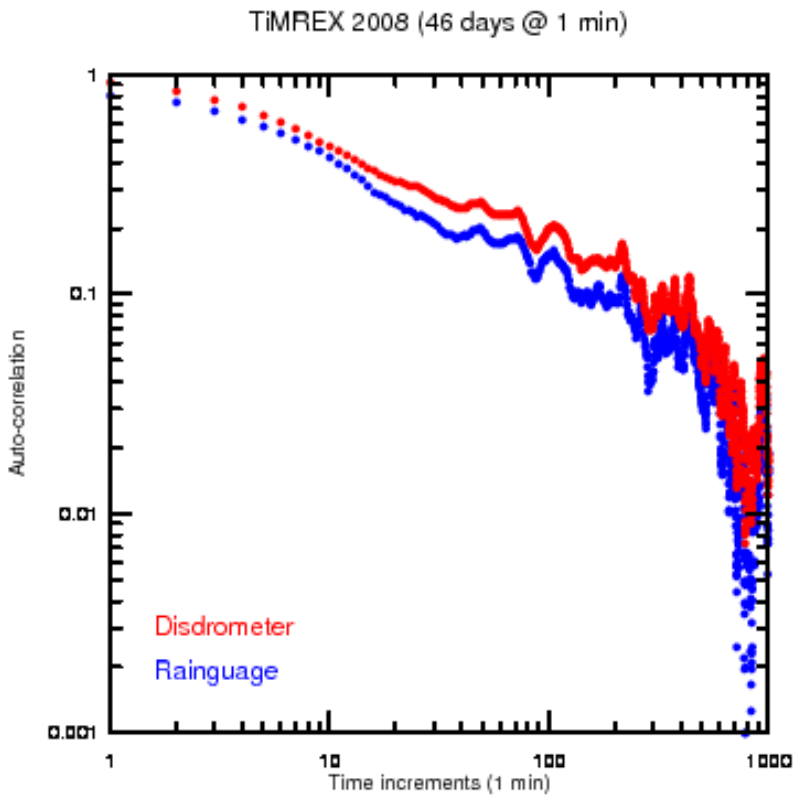


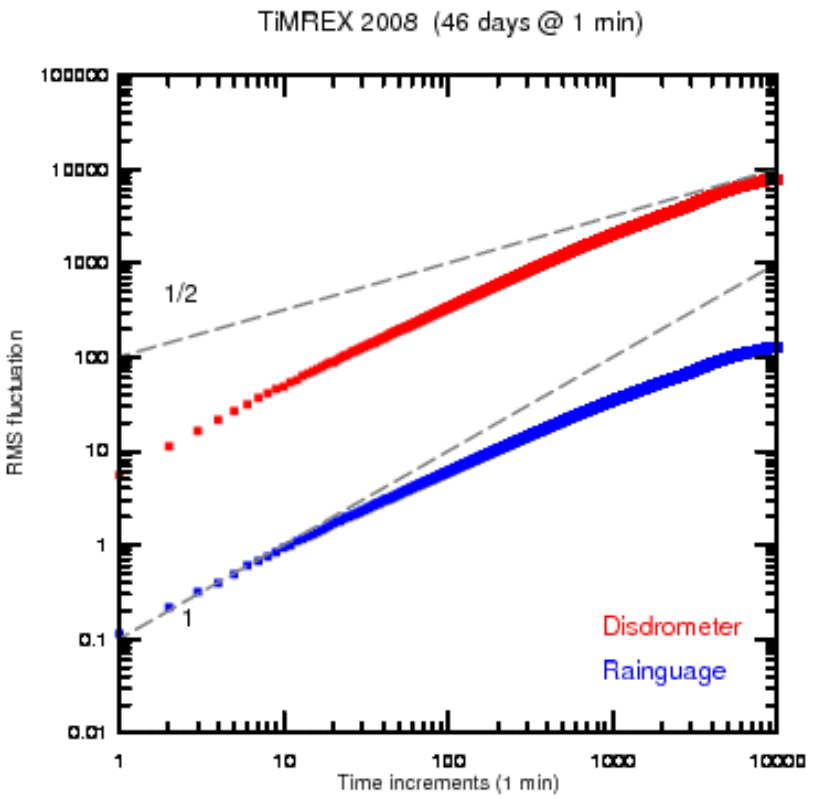




Autocorrelation







Autocorrelation

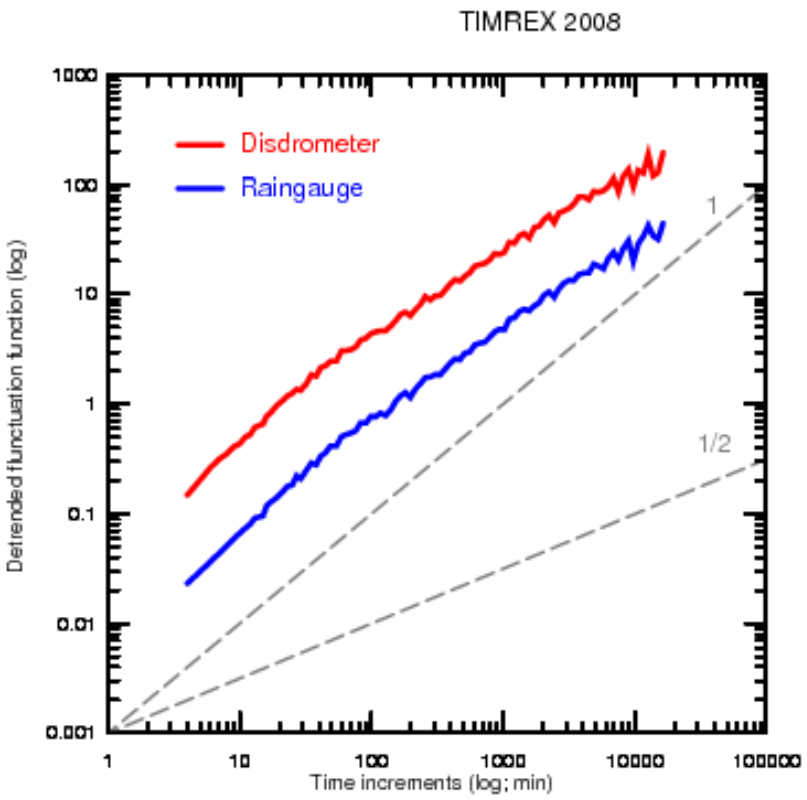
$$AC_n = \frac{1}{N}\sum_{k=1}^{N}\frac{\left(x(k)-\bar{x}\right)\left(x(k+n)-\bar{x}\right)}{\sigma^2}$$

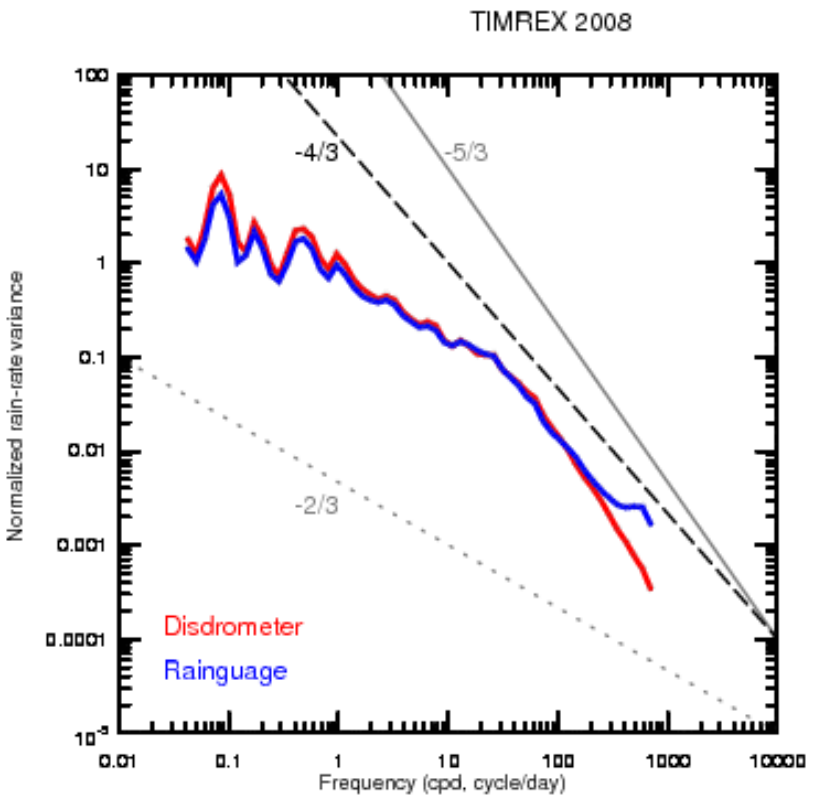
Fluctuation

$$y(k) \equiv \sum_{i=1}^{k} x(i)$$

$$F_n = \sqrt{\frac{1}{N}\left(\sum_{k=1}^{N}\left[y(k+n)-y(k)\right]^2\right)}$$

Long-Range Correlation ?





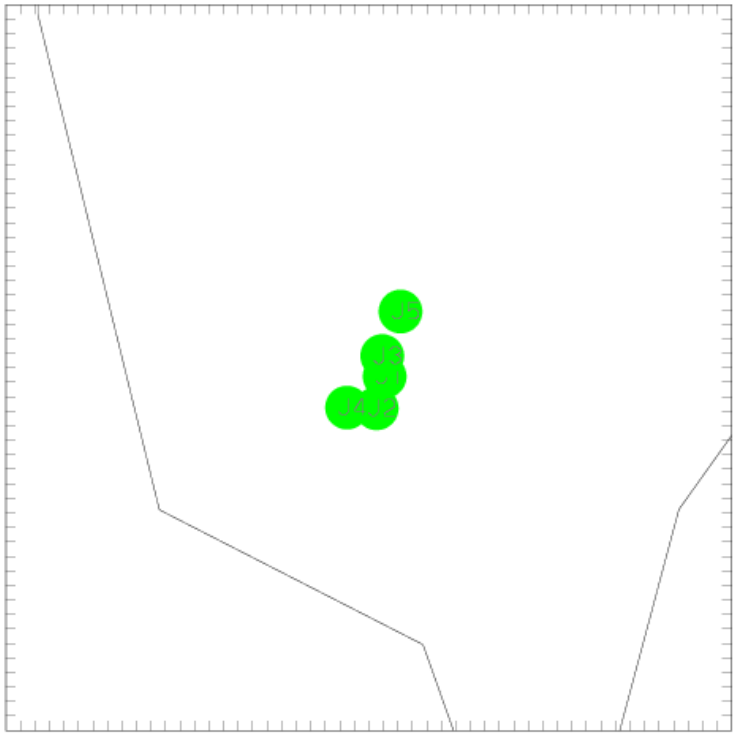
Detrended Fluctuation

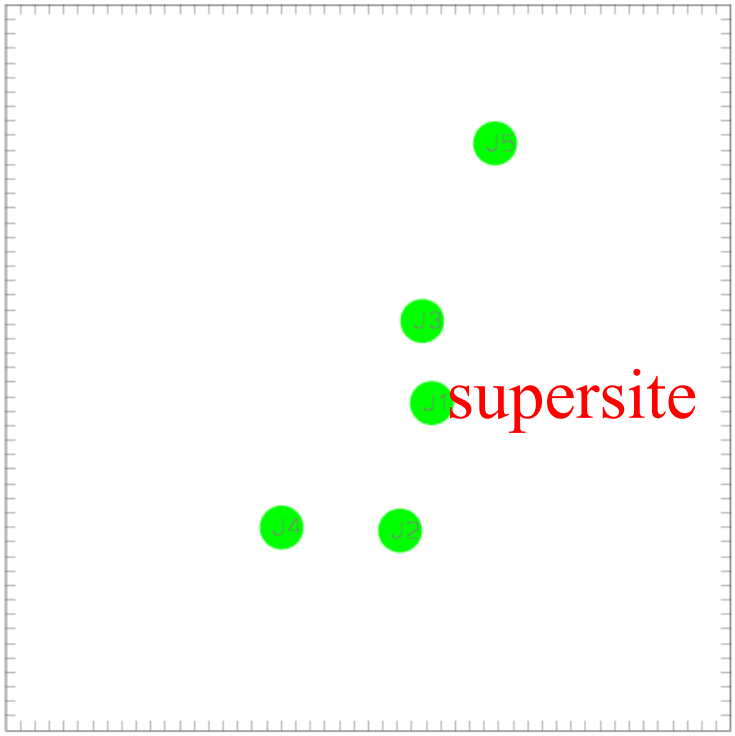
$$F(\Delta) \sim (\Delta)^{d}$$

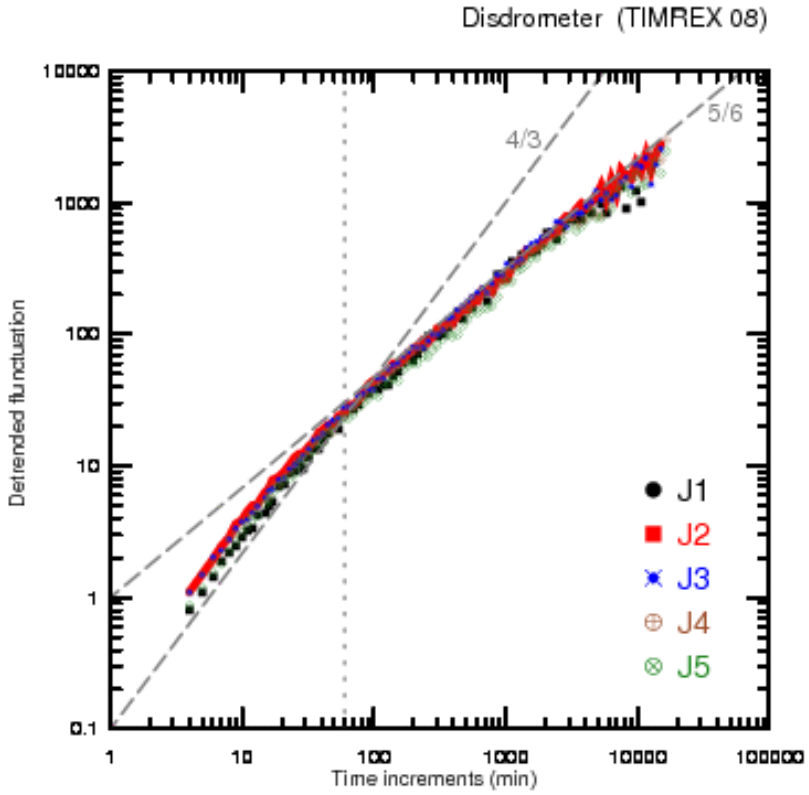
Temporal Spectra

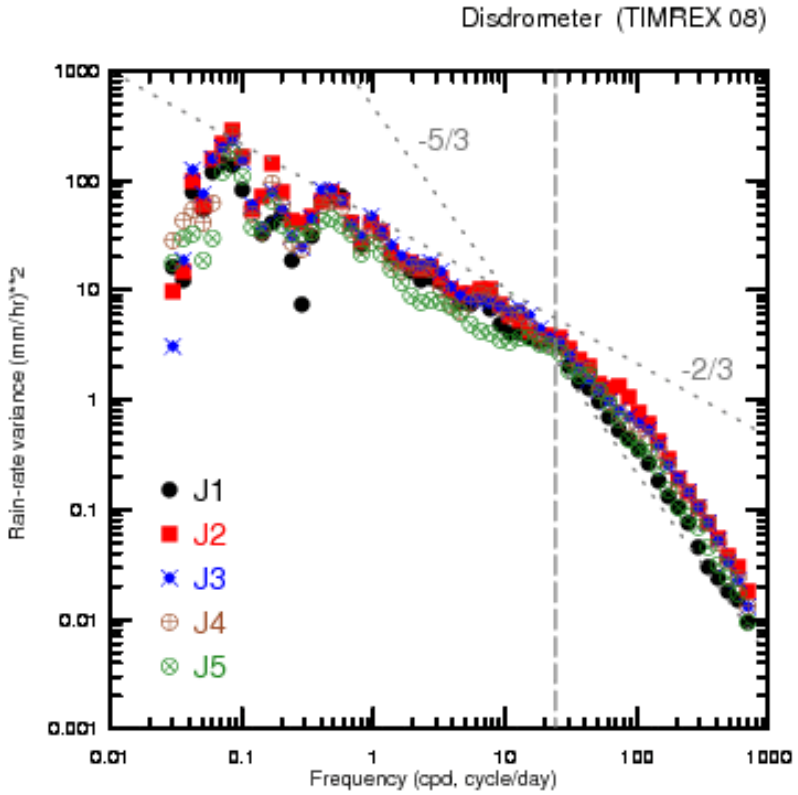
$$S(f) \sim (1/f)^{\beta}$$

Locations of disdrometer measurements









Detrended Fluctuation

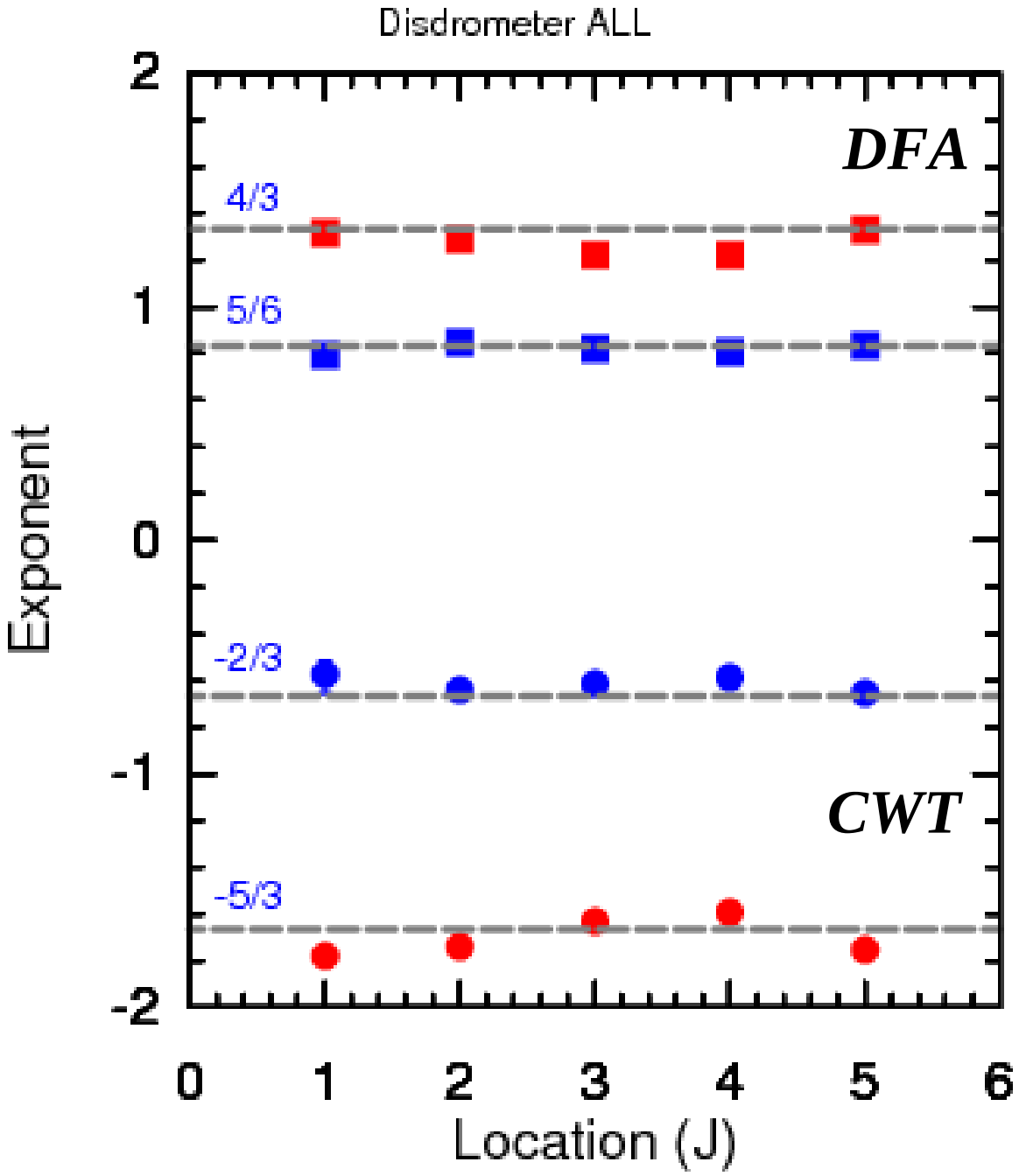
Temporal Spectra

$$F(\Delta) \sim (\Delta)^{d}$$

$$S(f) \sim (1/f)^{\beta}$$

1/f-noise

$$\beta = 2\,d - 1$$



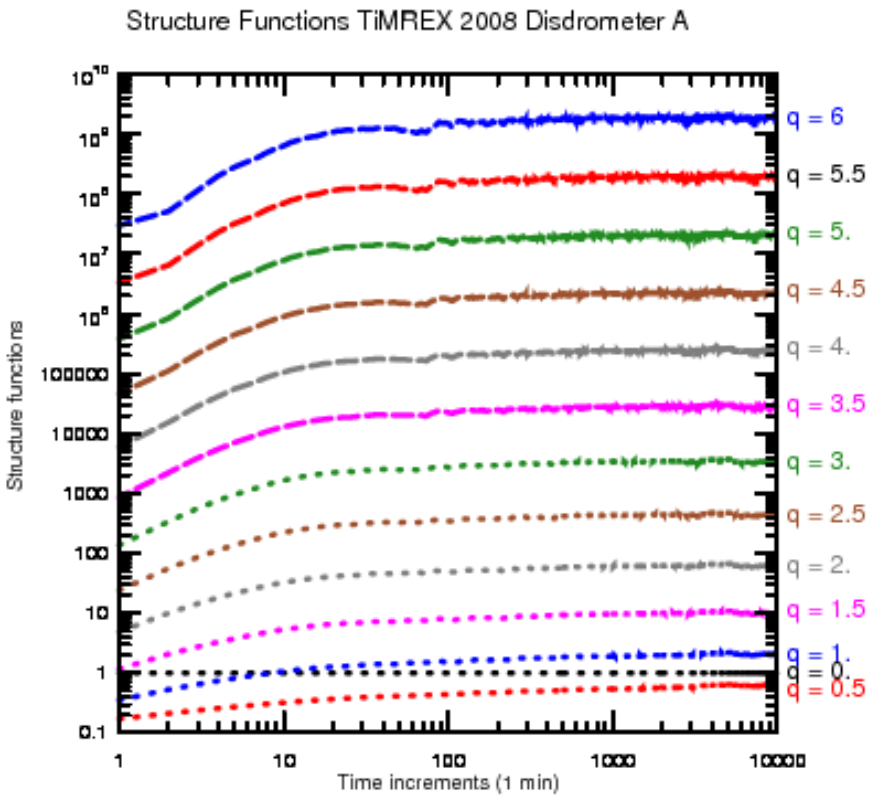
Consistent

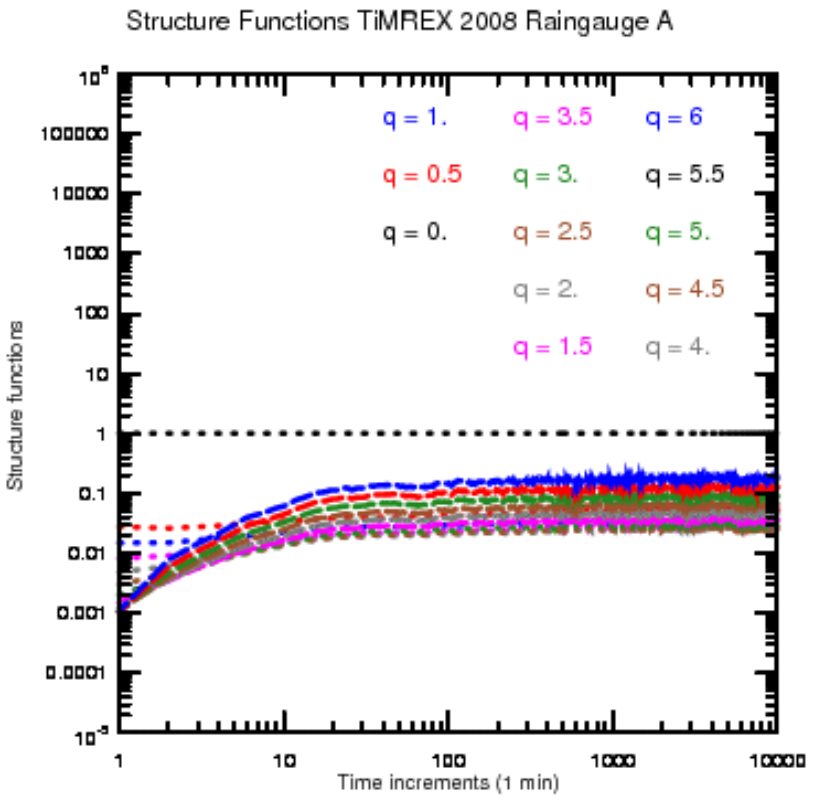
Dual-exponent

Scale Invariance

Fractals ?

$SF(\Delta_k) \sim < [\, x(i+\Delta_k) - x(i)\,]^q >$

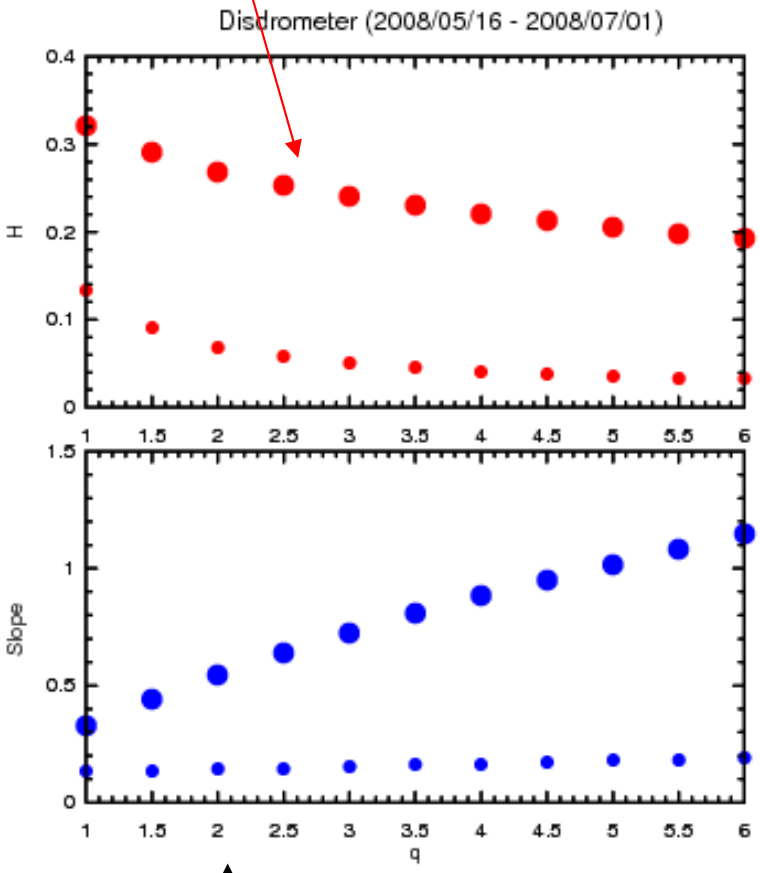


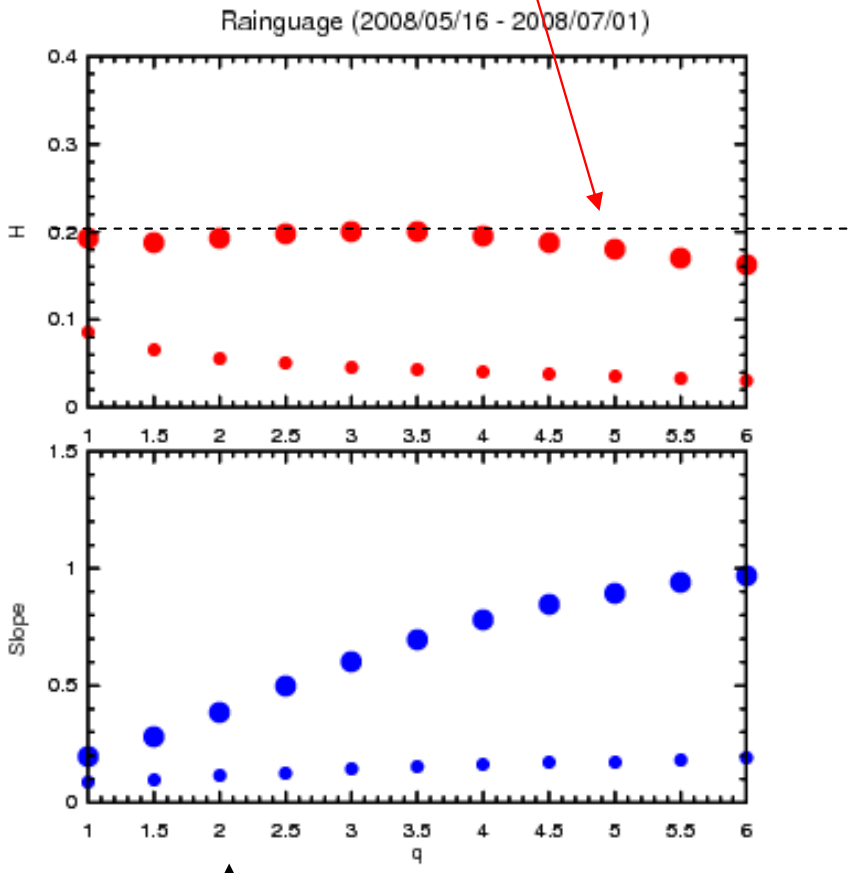


Multi-fractal

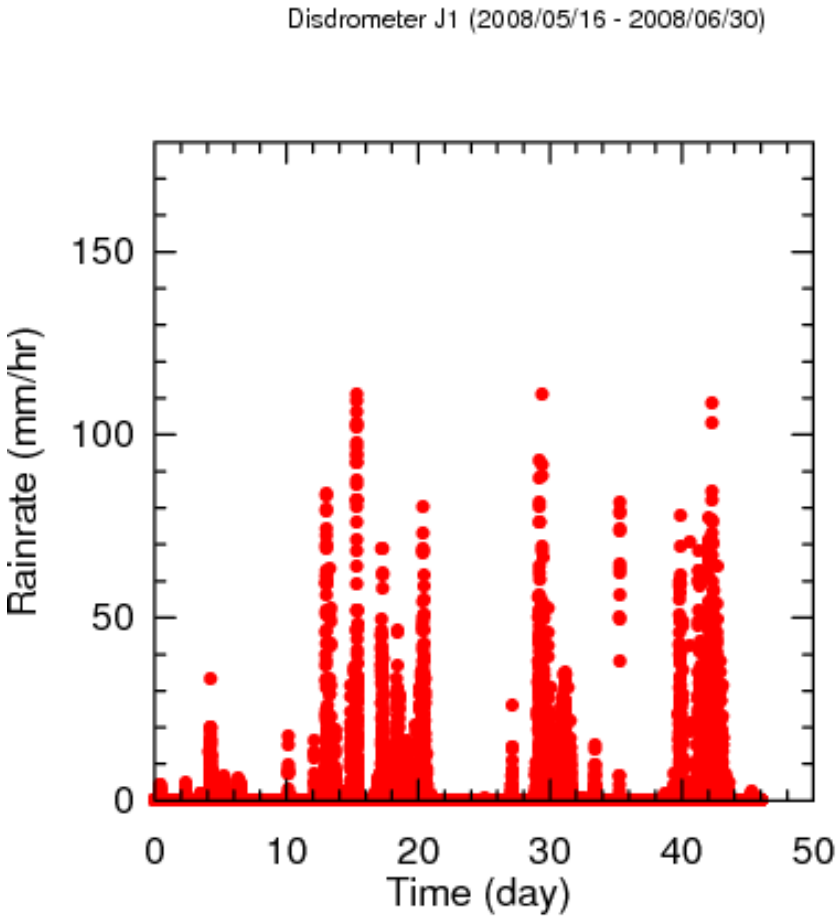
H=(slope)/q

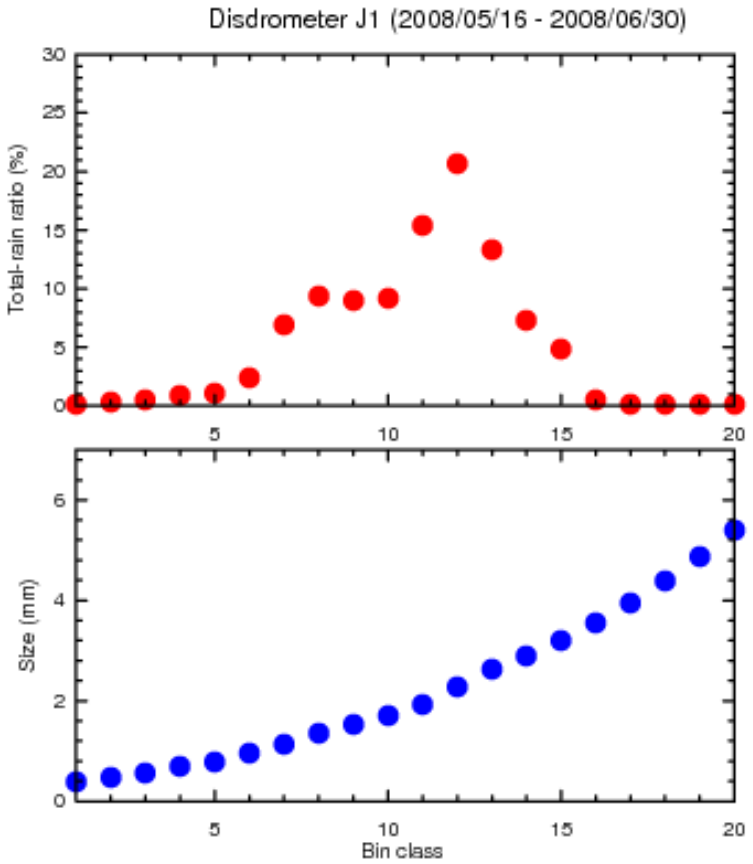
Mono-fractal

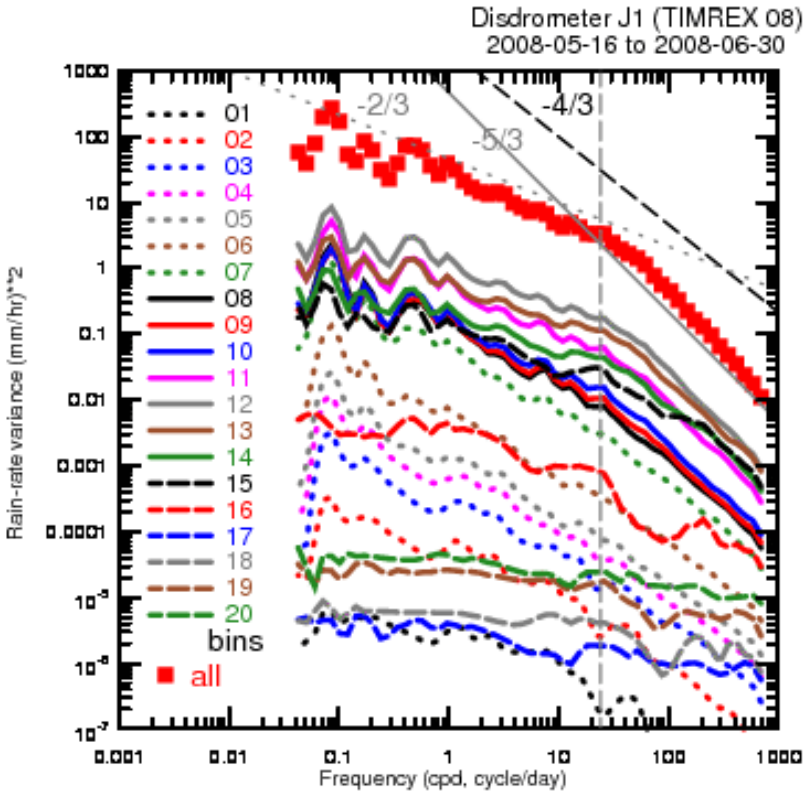


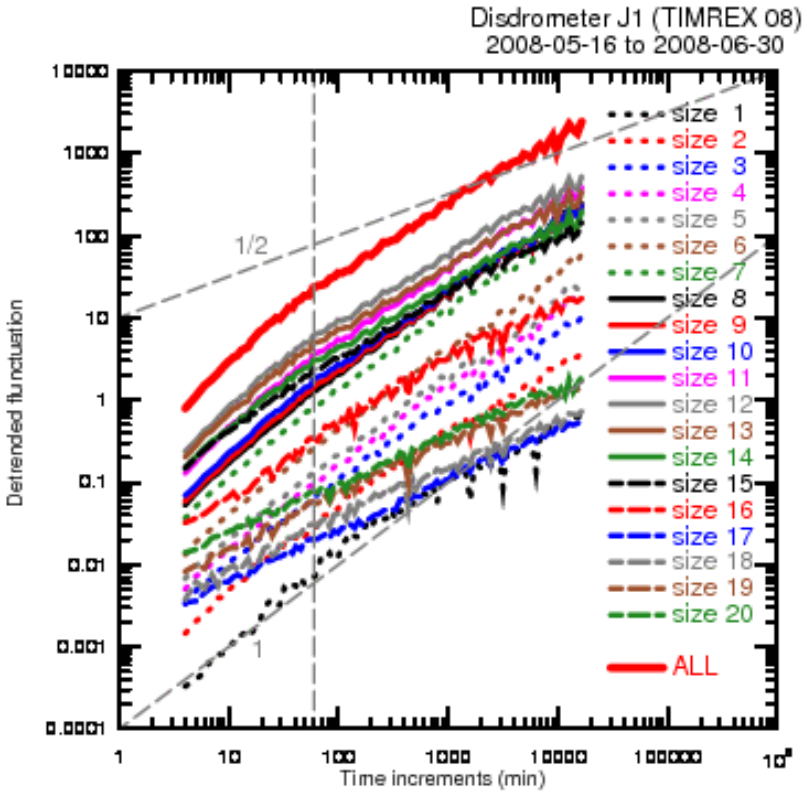


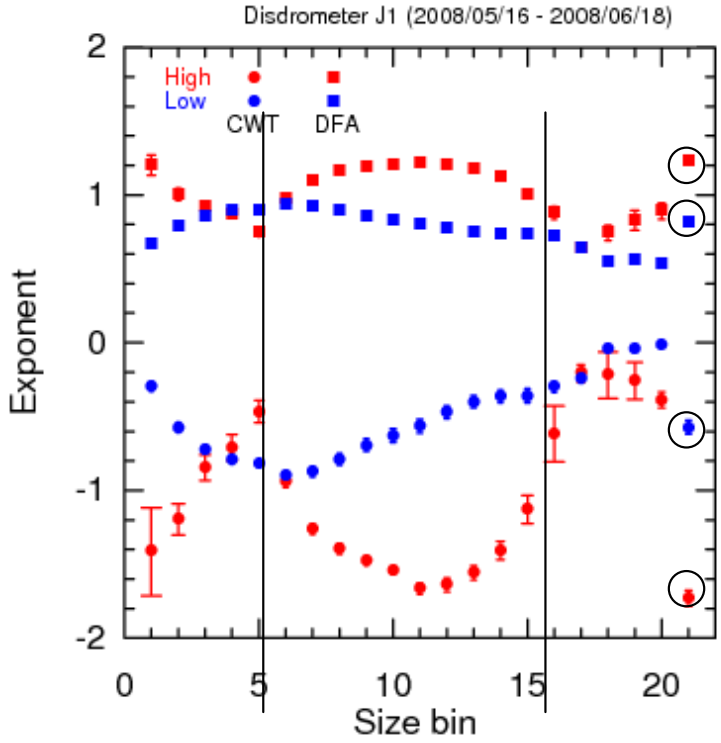
$$\beta = [\text{slope}(q=2)] + 1$$

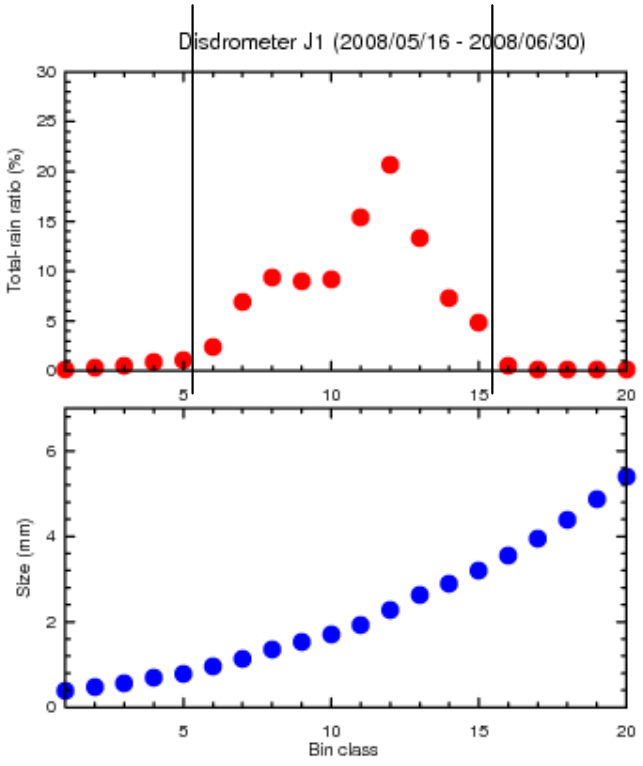

Disdrometer J1 (2008/05/16 - 2008/06/30)
Rainrate (mm/hr)
150
100
50
0
0
10
20
30
40
50
Time (day)


Disdrometer J1 (2008/05/16 - 2008/06/30)
Total-rain ratio (%)
30
25
20
15
10
5
0
5
10
15
20
Size (mm)
6
4
2
0
5
10
15
20
Bin class


Disdrometer J1 (TIMREX 08)
2008-05-16 to 2008-06-30
Rain-rate variance (mm/hr)**2
Frequency (cpd, cycle/day)
-2/3
-4/3
-5/3
01
02
03
04
05
06
07
08
09
10
11
12
13
14
15
16
17
18
19
20
bins
all


Disdrometer J1 (TIMREX 08)
2008-05-16 to 2008-06-30
Detrended fluctuation
Time increments (min)
1/2
1
size 1
size 2
size 3
size 4
size 5
size 6
size 7
size 8
size 9
size 10
size 11
size 12
size 13
size 14
size 15
size 16
size 17
size 18
size 19
size 20
ALL


Disdrometer J1 (2008/05/16 - 2008/06/18)
Exponent
Size bin
High
Low
CWT
DFA


Disdrometer J1 (2008/05/16 - 2008/06/30)
Total-rain ratio (%)
Size (mm)
Bin class

Octobe

Summary:

Both rainguage and disdrometer 1-min data indicate

- **Long-range correlation**
- **Dual-exponent scale invariance**

Possibly

- **Multi-fractals**
- **Self-organized criticality**